MAST31005 Algebra II exercise 3 (07.02.2024)

The following exercises concern ruler and compass constructions and origami constructions. You may use all of the elementary constructions presented in the lectures directly. That is, constructions 1-7 in the notes of 25.1. for ruler and compass, and Beloch's fold and folds 1-6 in the notes of 30.1. for origami.

1. Let $z \in \mathbb{R}$. Give a ruler and compass construction of z^2. Note that the construction of Proposition 3.11(iii) cannot be used immediately, since the triangle $\triangle(0, 1, z)$ is degenerate.

2. Let $z, w, u \in \mathbb{C}$ be distinct points.

(a) Give an origami construction of $v \in \mathbb{C}$ such that $|v - z| = |w - u|$.

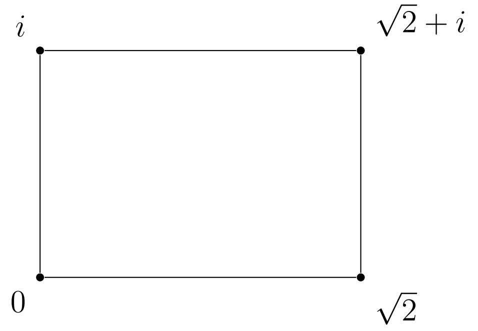
(b) Let ℓ be a line intersecting the circle $C(z, |w-u|)$ at P_1 and P_2. Show that the points P_1 and P_2 can be obtained as intersections of ℓ and folds using ℓ and the points z, v.

3. Let $z, w, u, v \in \mathbb{C}$ be distinct points such that the circles $C(z, |w-z|)$ and $C(u, |v-u|)$ intersect.

(a) Given an origami construction of the intersection points using z, w, u, v.

(b) Use (a) and exercise 2 to conclude that all constructible numbers are also origami numbers and that the origami numbers are a field.

In exercises 4–6, consider an A-series sheet of paper (e.g. A5) as a subset of $\mathbb{C}$ with the corner points and boundary lines already constructed:

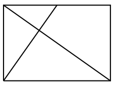


Let $A = \mathbb{Q}(\sqrt{2}, i)$ be the field containing these pre-constructed points.

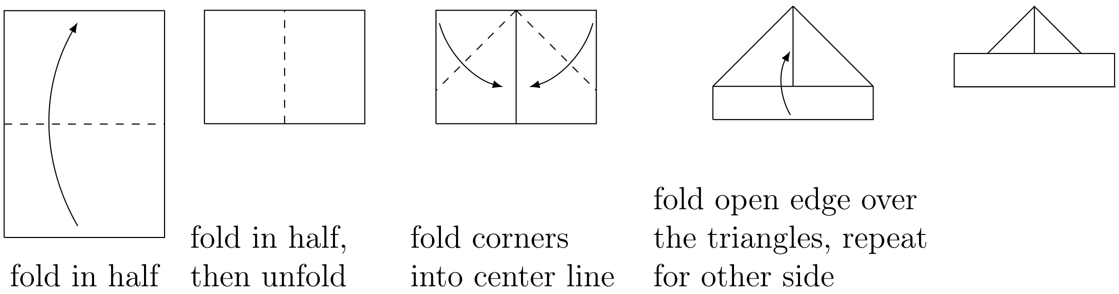
4. Construct $\sqrt{2}/3$.

(a) by ruler and compass

(b) by origami.

Hint: 

5. Consider the following origami hat construction:



(a) Repeat the construction as a mathematical origami without using folded states. How many extra folds are required?

(b) Let $H \subset \mathbb{C}$ be the set of all intersections of lines (including boundary lines) in the construction of (a). What is the degree $[\mathbb{Q}(H) : A]$?

6. Construct a point $\alpha \in \mathbb{C}$ such that $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 6$.